THE UNIVERSITY OF ALBERTA

A COMPARISON OF PROBLEM-SOLVING ABILITIES OF GRADE SIX STUDENTS IN TWO DIFFERENT MATHEMATICS PROGRAMS

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Comparison of Problem Solving Abilities of Grade Six Students in Two Different Mathematics Programs" submitted by Richard Alan Daly, in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

This study was part of the evaluation of Individually Prescribed Instruction (IPI) in Alberta schools. The purpose of this study was to compare the problem-solving abilities of grade six students who for two years had been exposed to an IPI mathematics program, with the problem-solving abilities of grade six students in conventional schools.

The sample used in this study consisted of seventy-eight grade six students selected from six elementary schools. Thirteen students were selected at random from each school. Three schools were IPI schools and three were control schools. Problem-solving proficiency was measured by an orally administered test, the Strategies of Problem Solving Test and by a written test, the Canadian Tests of Basic Skills (CTBS). The Strategies Test consisted of six mathematical problems which were unfamiliar to grade six students. The CTBS contains questions which are similar to many problems in grade six textbooks.

A subject's score on the Strategies Test was determined by the subject's ability to sense what was given and what was required in the problem, to predict a method of solution, and to verify a solution to the problem. A subject's score on the CTBS was the number of correct responses.

No differences existed amongst the three IPI schools with respect to scores achieved on either the Strategies Test or the CTBS.

It was found that the subjects in the control group scored significantly higher on the CTBS than did subjects in the IPI group. There was no significant difference between IPI and control groups with respect to achievement on the Strategies Test.

A subject's score on the Strategies Test was found to be significantly related to intelligence and his score on the CTBS.

On a test for interaction, it was found that neither mathematics program favored students in any of three intelligence levels.

From these findings the investigator concluded that IPI schools and control schools are equally effective in preparing students to solve unfamiliar and difficult mathematical problems. The control schools were found to be more effective in preparing students to solve the type of problems contained in grade six mathematics texts and standardized tests.

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CHAPTER ONE

THE PROBLEM, ITS NATURE AND SIGNIFICANCE

Most mathematics educators would agree that there is more to problem solving than obtaining a correct answer. Corle (1958) claims that "Pupils frequently get problems right without any real understanding of the meanings of the problem (p. 202)." The process or route used in finding the solution has long been recognized as an important aspect of the process of problem solving. The problem of evaluating these processes has never been clearly solved.

One method of analysing the process or strategy used is simply requiring students to "show all work" when solving a problem. The obvious disadvantage is that when students, usually the brightest, omit some steps, it is impossible to interpolate exactly, hence the strategy used is not evident.

Lazerte (1933) recognized this problem and devised an Envelope Test. As a student worked his way through a problem he opened envelopes which represented its partial and final solutions. The method of solution could then be traced by examining the opened evelopes. The obvious disadvantage is that the person constructing the test may not consider all the possible methods of solution. Some educators would consider this a very serious disadvantage. "Recognition should be given to the student who solves a problem in more than one way, and to the student who is able to find a particularly neat solution (Henderson and Pingry, 1953, p. 263)."

Others have developed techniques for identifying the strategies

used, by an oral interview. Weaver, 1955; Kilpatrick, 1967; Affolter, 1970). Affolter created a Strategies of Problem Solving Test in which she wanted to examine the relationship of language, tentative thinking an problem solving.

There is adequate justification for emphasizing the importance of problem solving.

> In most situations where "problem solving" is mentioned in mathematics, reference is being made to the ability to solve not only mathematical word problems, but also all other problems in which mathematics, might be used in the solution (Spitzer, 1967, p. 190).

On this basis Spitzer (1967) equates problem solving with thinking. Deese (1952) describes problem solving as a "supreme accomplishment of the evolution of mind (p. 254)." Gagne (1967) rates problem solving as the highest form of learning, and that during the process of problem solving, students instruct themselves to select adequate strategies to use in mastering the problem.

Problem solving appears to deserve an integral part of education.

> The principal goal of education is to create men who are capable of doing new things, not simply of repeating what other generations have done - men who are creative, inventive, and discoverers. The second goal of education is to form minds which can be critical, can verify, and not accept everything they are offered (Elkind, 1968, p. 80).

If strategies of problem solving are important, we then must wonder if strategies can be taught in the classroom. Buswell (1956) states "The evidence gives no support to the notion that problem solving must follow the neat, precise recipes that are so frequently encountered in textbooks on methodology (p. 133)."

This seems to imply that each child may find a different strategy or sequence of strategies to be best suited for his own use. If a teacher attempted to teach a class as a whole, many of the students would not have their needs met. Lindvall and Bolvin (1967) claim that "Learning

takes place only on an individual basis; consequently instruction must be individualized (p. 233)."

In September, 1969, a pilot project was established to study the suitability of an Individually Prescribed Instruction (IPI) program in mathematics as curriculum for Alberta schools. The Alberta Human Resources Research Council selected three elementary schools and replaced the present mathematics program with an IPI program of mathematics.

If this program is to meet the needs of elementary school children in Alberta, it seems reasonable that children should develop useful problem-solving strategies. At this time research is needed to evaluate all aspects of the IPI program as it applies to Alberta schools, including processes of problem solving.

I. PURPOSE OF THE STUDY

The purpose of this study was to ascertain the problem-solving abilities of grade six children using the IPI mathematics program and to compare these with the problem-solving abilities of grade six children using the conventional mathematics program. Specifically, the research was intended to answer the following questions:

1. Is there a difference between IPI schools and control schools with respect to the process of solving problems and achievement on a standardized problem-solving test?
2. Is there a difference between the three IPI schools with respect to the process of solving problems and achievement on a standardized problem-solving test?
3. Is there a correlation between the process of solving problems and achievement, intelligence, and age?
4. Does any I.Q. group receive preferential treatment in either IPI

schools or control schools?

II. DEFINITION OF TERMS

Verbal Problem of Arithmetic - the word description of a quantitative situation about which a question is asked and which does not indicate the operation(s) to be used. (Spitzer, 1948, p. 180).

IPI Schools - the schools and students who are involved in the experiments in Individually Prescribed Instruction. They are Forest Heights in Edmonton, St. Vincent De Paul in Calgary, and Millarville in the Foothills School Division.

Control Schools - those schools selected by the Alberta Human Resources Research Council to serve as controls for the IPI schools. They are Princeton in Edmonton, St. Leo in Calgary, and Red Deer Lake in the Foothills School Division.

Strategies of Problem Solving - "the process of orienting oneself to a problem, predicting or suggesting possible solutions and verifying the result." (Affolter, 1970, p. 4)

Problem-Solving Ability - the subject's score on the Strategies of Problem-Solving Test.

Arithmetic Problem-Solving Achievement - the subject's score on the Arithmetic Problem-Solving Subtest of the Canadian Tests of Basic Skills.

Intelligence - the subject's I.Q. on the Lorge-Thorndike Tests of Intelligence. Both verbal and non-verbal scores were recorded.

Individually Prescribed Instruction - is an innovation in the organization of instructional materials and procedures that is designed to permit pupils to progress at individual rates in mastering the sequence of objectives in given content areas (Yeager and Lindvall, 1968).

The "Above Average", and "Below Average" Pupil - for the pupose of this

study, these terms are defined in terms of non-verbal intelligence quotients. "Above average" describes students in the I.Q. range of 120 to 150 while "average" describes students in the range of 100 to 119, and "below average" describes students in the range 71 to 99.

III. THE HYPOTHESES

The null hypotheses tested were as follows:

1. There is no significant difference between the IPI groups and the control groups with respect to:
   (a) problem-solving ability as measured by an oral test
   (b) verbal problem-solving achievement as measured by a written test.
2. There is no significant difference among the three IPI schools with respect to:
   (a) problem-solving ability
   (b) verbal problem-solving achievement.
3. There is no significant relationship between problem-solving ability as measured by an oral test and:
   (a) verbal problem-solving achievement
   (b) intelligence
   (c) age.
4. There is no significant interaction between pupil intelligence and type of school with respect to:
   (a) problem-solving ability
   (b) verbal problem-solving achievement.

IV. DESIGN OF THE STUDY

The population for this study consisted of three IPI schools and

three control schools as listed in Definitions. There were 58 students in Forest Heights, 48 students in St. Vincent De Paul, 18 students in Millarville, 70 students in Princeton, 44 students in St. Leo, and 28 students in Red Deer Lake.

The sample used in the study consisted of 13 students randomly selected from each of the six schools. There were, then, 39 students in the IPI group and 39 students in the control group.

During the oral interview, a Strategies of Problem Solving Test constructed by Affolter (1970) and refined by the investigator, was administered to each of the 78 students in the sample. The interviews were tape recorded and subsequently scored.

It was also necessary to gather intelligence scores as measured by the Canadian Lorge-Thorndike Tests of Intelligence, age scores in months, verbal problem-solving achievement as measured by the Arithmetic Problem-Solving Subtest of the Canadian Tests of Basic Skills, and verbal problem-solving achievement as measured by the same test before the IPI experiment began.

Hypotheses 1 and 2 were tested using Multiple Linear Regression Analyses. Hypotheses 3 was tested using a Pearson-product moment correlation. Hypotheses 4 was tested by means of two-way analysis of variance.

V. BASIC ASSUMPTIONS

The investigation was based on the following assumptions:

1. It was assumed that the Strategies of Problem Solving Test would measure problem-solving abilities of the students tested.
2. It was measured that the problem-solving strategies measured were a representative sample of the strategies used by all students in the population from which they were selected.

VI. LIMITATIONS

The investigation had the following limitations:

1. Since the schools concerned had previously been selected by the Human Resources Research Council, the general location of population schools could not be considered.
2. Variables that may affect problem solving, such as reading ability, attitudes toward arithmetic, computation ability, sex, and motivation level, were not controlled in this study.
3. This study is pertinent to the six schools being investigated and results may not apply to other IPI projects or other programs of an individualized nature.

VII. SIGNIFICANCE OF THE STUDY

1. With the present emphasis on the processes of learning, rather than the products of learning, more research is needed to determine methods of identifying these processes.
2. There is a need to evaluate educational innovations; this study identifies the IPI program as an innovation in need of evaluation.

VIII. ORGANIZATION OF THE REPORT

The problem has been introduced in Chapter One. Chapter Two contains discussions on the nature of verbal problems, acquiring abilities to solve verbal problems, the process of problem solving, and individualized instruction. A summary of the relevant research concludes the chapter. Chapter Three describes the design of the study, the sample, the instrumentation and the analysis of data. Chapter Four summarizes the results of the data analysis. Finally, Chapter Five describes the data analysis, interpretations of the analysis, a summary of the study, conclusions and implications for research.

CHAPTER TWO

REVIEW OF RELATED RESEARCH AND LITERATURE

Since the beginning of this century problem solving has been the most popular topic for research investigation in elementary school mathematics (Suydam, 1967). One possible explanation might be the high status given problem solving by most teachers. Some studies have attempted to outline the sequential steps used by a subject involved in problem solving, while many studies have compared different techniques of teaching or improving problem-solving abilities. Many studies have yielded conflicting results, particularly in terms of indicating the classroom conditions which provide for optimum growth in problem-solving abilities.

Recently, individualized instruction has been a topic of frequent discussion and has been widely experimented with. In most subject areas materials have been constructed for use in individualized classrooms. Mathematics is no exception. Although most of the mathematics projects in individualized instruction have been concerned with mathematics in general, some have dealt specifically with problem solving.

The purpose of this chapter is to review literature related to problem solving and individualized instruction. The first section of the chapter contains a discussion of what constitutes a verbal problem of arithmetic. The second section examines the process of problem solving under the subsections of the process, developing and improving problem-solving abilities, and analysing the process of problem solving. The third section contains a review of literature relative to individualized instruction in

mathematics with particular reference to IPI.

I. THE VERBAL PROBLEM OF ARITHMETIC

Educators have never unanimously agreed on a definition for a verbal arithmetical situation stated in words which required the pupil to first decide which operation to perform. He limited the true arithmetic problem to those problems which arose out of experience in which children had engaged in actual concrete situations. This eliminated from the class of true arithmetic problems many problems written in text books. The printed material should only be used, he believed, as a supplement to the actual experience in which children became involved.

Wheat (1937) considered a situation to be true problem only when doubt arose about the way in which the situation should be handled. Only when there was ignorance or complete uncertainty of a method of solution of a quantitative situation did a problem exist.

This view was later supported by Cronbach (1948). Cronbach stated, ". . . it is not posing the question that makes the problem, but the person's accepting it as something he must try to solve (p. 34)." It did not matter who posed the problem; the crucial issue was the extent to which the child's ego became involved in the problem.

Later still, Henderson and Pingry (1953) defined a verbal problem of arithmetic in terms of the conditions which are necessary for the process of problem solving to take place. The conditions are:

1. The individual has a clearly defined goal of which he is consciously aware and whose attainment he desires.
2. Blocking of the path toward the goal occurs, and the individual's fixed patterns of behavior or habitual responses are not sufficient for removing the block.
3. Deliberation takes place. The individual becomes aware of the problem, defines it more or less clearly, identifies various possible solutions, and tests these for feasibility (p. 230).

When these three conditions were met, a problem existed for the child.

Many educators accept a combination of Henderson and Pingry's and Cronbach's definitions. Marks, Purdy and Kinney (1965) claim that a verbal problem exists whenever a student is attempting to solve a quantitative situation which requires the use of abilities which he has not made automatic. They add one other dimension as well to the definition. A verbal problem must be an application of mathematics to a life situation.

A very similar definition was given by Fehr and Phillips (1967). They defined a verbal problem as "a situation for which no ready-made procedure toward a solution is available to the person facing such a situation (p. 375)."

The most vital characteristic of a verbal problem, it seems, is that the individual does not possess an automatic, ready-made, response by which a solution may be attained. "If a pupil produces an appropriation response from habit, or sees no relationship in the conditions, there _is_ no problem for him (Herlihy, 1964, p. 308)."

It is useful to keep such characteristics in mind, but simply describing some necessary conditions which must be met before a verbal problem exists is not the most useful definition to use when attempting to classify a situation as a verbal problem. Spitzer wrote that a

> Verbal problem, as used here, refers to the word description of a quantitative situation about which a question is asked. The solution of a verbal arithmetic problem requires the use of one or more of the fundamental arithmetical processes (Spitzer, 1948, p. 180).

In summary, we may conclude that a verbal problem is a quantitative situation, the solution of which requires some reflective thought and an element of uncertainty.

II. THE PROCESS OF PROBLEM SOLVING

The Process

How we think and solve problems has been a popular topic in philosophical, psychological, and educational spheres since the time of Plato.

> . . . problem solving is a set of events in which human beings use principles to achieve some goal. When a problem solution is achieved something is also learned, in the sense that the individual's capacity is more or less permanently changed. What emerges from problem solving is a higher-order principle, which thereupon becomes a part of the individual's repertory. The same class of situation, when encountered again, may be responded to with great facility by means of recall, and is no longer looked upon as a "problem". Problem solving, then, must be definitely be considered a form of learning (Gagne, 1967, p. 157).

Problem solving, or reflective thought was described by Dewey (1933) as a five stage process.

The first stage, the problem presenting situation, occurs when an individual is in a situation that is not familiar to him. His previous experience cannot give him immediate satisfaction. Thinking begins when the answer is desired, but not known, by the individual.

The next step is the analysis or examination, within the mind, of the situation in which there is dissatisfaction. He discovers why he is dissatisfied and states his problem.

The third element of thinking is a search for leads and tentative hypotheses. The cycle of framing and testing hypotheses until a method of solution is reached is the heart of the problem-solving process.

Dewey's fourth phase is deduction. Deduction involves organizing the solution of the problem into some frame of reference. If learning is to take place it is necessary, he says, to emphasize the relationship of the solution to some overall structure.

The final stage is a verification or action process. It is to

a large degree a reapplication of the first four steps in a new problem.

Dewey's explanation has remained largely unchallenged. Others have attempted to create an explanation of their own, but most have not criticized Dewey. They could not criticize Dewey because their theories were very similar.

Polya (1957) suggests that the process of problem solving is aptly described by four stages. An individual first attempts to understand the problem, then develops a plan, carries out the plan, and finally checks the result. Polya and Dewey differ only with regard to the inclusion of the problem-presenting situation as a stage in the process of problem-solving.

Merrifield (1960) suggests that problem solving is a cyclic procedure consisting of five phases. The stages, he claims, are preparation, analysis, production, verification and reapplication. Klausmeier and Loughlin (1961) concurred with Merrifield in outlining the method used by one student:

> Richard (a) accepted the task as a problem and set a goal of achieving a correct solution, (b) analyzed the available information in relation to his goal, (c) produced a solution and independently rejected it (verification), (d) applied knowledge from this and subsequent solutions to each successive solution produced (re-application), and (e) finally verified and accepted the correct one (p. 150).

Polya (1965) later suggests that the essence of problem-solving is the two stages "GUESS AND TEST" (p. 156).

Affolter (1970) decided that three stages were sufficient to describe the process of problem solving. She named these categories sensing, predicting, and verification.

> Ideally, for each problem posed, three things should occur after the individual has become aware of the problem as something he would like to try and solve: he defines the problem or <u>senses</u> the situation or conditions imposed by it; he identifies various hypotheses (<u>predicts</u>)

> that could lead to a solution; and he verifies or tests the hypotheses for feasibility (Affolter, 1970, p. 33).

It seems futile to argue about names of phases, or in fact the number of phases used by individuals engaged in problem solving. The number of phases depends entirely on a particular definition of each phase.

Few educators will suggest that problem solving must follow any number of specific stages explicitly. Problem solving, clearly, is an individual experience.

> The evidence gives us no support to the notion that problem solving must follow the neat, precise recipes that are so frequently encountered in textbooks on methodology (Buswell, 1956, p. 133).

> The major finding of this part of the study was that, although some agreement in patterns of thinking was revealed, the most striking characteristic was variety, rather than similarity, in sequence of thinking (Buswell, 1956, p. 136).

Most educators will agree that, regardless of title, several phases do exist in a problem-solving situation.

> "But all the descriptions agree on one essential feature: that there is a cycle involving the formulation of a testing procedure or trial, the operation of this testing procedure and the comparison of the results of the test with some criterion (Bruner, 1966, p. 50)."

Stated in similar terms:

> "Whether a hypotheses is effective or ineffective in solving a problem can be determined either by actually testing it out or by elaborating its consequences and implications in an attempt to uncover any error in fact or inconsistency with tested knowledge (Henderson and Pingry, 1953, p. 245."

Which of these phases is the most important, is however, not so universally agreed upon. Doty (1940) concluded,

> "that almost the entire process of solving centers in the decision as to the mathematical process(es) to be used. Success at this point requires understanding both of mathematical processes and of the language which signifies need for their use (p. 162)."

An examination of 2,778 grade six children's attempts at problem solving revealed 50 errors in selecting what was wanted in the problem, 674 failures to recognize and indicate the relationships and operations essential to the correct solution, and 402 cases in which computation was in error (Clark and Vincent, 1971).

Somewhat in opposition to the above findings, Mitchell (1932), carried out a study in which the five most difficult problems on a grade seven test were given again on a subsequent test to which were added analytical questions about the above problems. The analytical questions directed the student's attention to key parts of the question. The subsequent test showed a 180% gain in performance scores on these five questions. He concluded that understanding what was asked as well as what intermediate steps must be involved in finding the final answer were the phases of problem solving most vital to success.

There is some evidence to show that the method or process used is somewhat influenced by the ability of the individual.

> A close study of how pupils attack arithmetic problems shows two general types of approaches. On the part of the dull pupils we find a mechanical, thoughtless procedure, while the bright pupils make a quick plunge into the middle of the problem, often skipping the opening step or two. Sometimes the result is found mentally, but often the pupil is unable to set his work down on paper (Rimer, Wilson and Knight, 1924, p. 274-277).

Affolter (1970) found supportive evidence for this proposition. She found that a poor problem solver was reluctant to try an alternate method of solving a problem even when he was convinced that his answer was wrong. He thoughtlessly manipulated numbers, primarily involved with the computation process. Good problem solvers engaged in what Affolter called "tentative thinking" in which the student suspended judgment as to the correctness of the method until the solution was verified.

Bruner (1962) gives further support:

> Intuitive thinking, the training of hunches, is a much-neglected and essential feature of productive thinking not only in formal academic disciplines but also in everyday life. The shrewd leap to a tentative conclusion--these are the most valuable coin of the thinker at work, whatever his line of work. Can school children be led to master this gift? (p. 13-14).

To compound the dilemma of how many phases exist in the problem-solving process, Brian (1966) suggests that there is not one, but four processes of mathematics. He lists them:

1. The process of constructing mathematical models.
2. The process of conjecturing . . .
3. The process of settling conjectures as being either true or false . . .
4. The process of using known or given axioms and theorems on problems where they clearly apply (p. 3-6).

Brian was convinced that traditional mathematics programs are designed to teach the use of the fourth process only. He then designed an experiment which attempted to teach the first three processes. The treatment involved the use of flow charts with a conjecture loop and a verification loop. He found that subjects could acquire some ability to use the first three processes when they had not previously been able to do so. He concluded that guessing and intuition can play vital roles in solving problems which are unfamiliar to the learner.

For purposes of clarification, neither Spitzer (1948), Herlihy (1964), nor Gagne (1967) would accept Brian's fourth process as being engaged in true problem solving. They would reject the fourth process on the basis that it produces appropriate responses from habit.

Developing and Improving Problem-Solving Abilities

A logical question to ask is whether you can actually teach a process of problem solving or whether the process is something that simply develops, as does thinking. Stevenson (1924) reported that training children

to analyse problems for several weeks resulted in marked improvement in their ability to solve problems.

Studies have been carried out to compare different techniques of teaching problem solving. Washburne and Osborne (1926) agreed in part with Stevenson. They carried out a study with 763 grade six and seven children exposed to three different treatments. One group was simply given a large number of problems to solve, using no special technique. The children in the second group were instructed to thoroughly analyse the problem: (a) read carefully, (b) determine what is to be found, (c) determine what elements in the problem will help find the answer, (d) decide what process to use, (e) estimate roughly the magnitude of the results, and (f) solve the problem. The third group were trained to see the analogy or similarity between difficult written problems and corresponding easy problems, and thereby to decide what process to use in attacking the difficult problems. Their findings showed that while all three groups showed improvement, the group which solved many problems was the superior group.

Hanna (1929) compared a conventional formula method of teaching problem solving, a graphical method outlining the steps used by a tree diagram, and an individual-use-any-method. He found the conventional formula method to be the poorest method and no difference between the other two.

A reasonable guide to follow when teaching the process of problem solving would seem to be:

> First, the teacher needs to see that children are given many opportunities to solve problems . . .
> Secondly, children should be allowed to solve problems in various ways . . .
> Lastly the teacher should provide for the development of understandings of the four fundamental processes, since this is a vital factor as

> far as improvement in problem solving is concerned . . . Children should be asked constantly to explain how a problem is to be solved and why a particular process is thought to be appropriate (Pace, 1961, 231-3).

A review of research on teaching problem-solving abilities leaves a reader somewhat uncertain.

> In spite of all the proposals and research it is probably not too far amiss to summarize the results of present-day research by the single statement: The best way to teach children how to solve problems is to give them lots of problems to solve (Van Engen, 1959, p. 74).

Brown and Abell (1965) summarize research in a slightly different way. They propose that there seems to be no one method of teaching which is best for all students, and that this may be due to different maturation rates.

Research seems to indicate that there are specific practices which will improve the ability to solve problems regardless of what method is being used.

Spitzer and Flournoy (1956) analysed the techniques for improving problem solving (17 in all) suggested by five textbook series. They reported:

> "Use of drawings in solutions, pupil formulation of problems, and oral explanation of solutions by pupils were not popular in these five textbooks. Still more surprising is the fact that not a single specific procedure for improving problem solving is recommended by all five books (Spitzer and Flournoy, 1956, p. 181-2)."

Oral explanations of solutions are, however, recommended by at least two educators. Corle (1958) claims that many students get the right answer without understanding the problem and therefore,

> Teachers must challenge problem solutions more often, use extraneous materials, and compel students to justify methods by concrete examples. Questioning students engaged in oral problem solving revealed improvement in their rationale (Miller, 1960)."

Deliberate attempts to improve problem-solving abilities have been

met with only partial success. Students with I.Q. scores less than 90 seem to benefit most from a remedial program (Stevenson, 1924). In a similar attempt, students with I.Q. scores lower than 100 benefited most from a program based on systematic instruction of the reading of verbal problems (Monroe and Engelhardt, 1933). Johnson (1944) reviewed the literature on problem solving in arithmetic and found that research had not shown that "teaching a method of problem solving improves ability to solve problems (p. 482)." Post (1967) similarly concluded that special study of a structure of the problem-solving process appears not to enhance the problem-solving ability of seventh grade students.

There is some evidence to suggest that it is beneficial to structure problem-solving lessons of multilevel difficulty (Riedesel, 1964). He suggests that a child can improve his problem-solving abilities if he is given problems to solve which vary in terms of difficulty and application. This variety can be partially accomplished by using very old textbooks and foreign textbooks.

Factors which seem to affect problem-solving abilities are intelligence (Rimer, 1924; Klausmeier and Loughlin, 1961; Post, 1967; Affolter, 1970), age (Lazerte, 1933), and simply what goes on in the classroom (Kilpatrick, 1967). Corle (1958) found a significant relationship between problem solving and each of the following: concept formation, reasoning ability, confidence in computation ability, and understanding of arithmetic vocabulary. Chase (1960) concluded that the three factors of primary concern in predicting problem-solving ability were: the ability to compute, the ability to read and note details, and the possession of fundamental knowledge of the generalizations which underlie the number system. Post (1967) and Affolter (1970) concluded that sex is not a

significant factor in determining problem-solving abilities.

Analyzing the Process of Problem Solving

It has long been recognized that students in a given classroom will use a variety of means and processes to solve the same verbal problem of arithmetic. In 1928, Monroe studied the problem-solving behaviors of over 9000 students in grades six to eight. He found that a large percentage of these students made no attempt to reason when confronted with a problem. They simply relied on habit, experiencing success only with familiar problems. Many did not even attempt problems that were expressed in unfamiliar terms. Cronbach (1948) would say that for these pupils no problem existed.

A few years later, Lazerte (1933) created his Envelope Test, which was an attempt to trace the sequence of choices made by an individual in the process of solving a problem. He found that difficult problems encouraged the use of trial and error procedures. He also supported the belief that students vary greatly in their choice of approach to problem solving.

Some twenty years later, strategies used in the process of problem solving began to be investigated on the basis of data gained during an oral interview. Corle (1958) had grade six students "think-aloud" when solving arithmetic problems. Corle interviewed 74 students, while solving eight problems, and came to the following conclusions:

1. A pupil's idea of what a problem means is important to him . . .
2. Word and number clues served as the predominant method of attack in problem solving . . .
3. Pupils tend to be overconfident about their arithmetic success. This undue assurance indicates over-dependence upon computation as a tool for problem-solving.
4. Understanding the terms used in arithmetic is a definite factor in problem solving efficiency . . .

5. The greatest number of incorrect solutions occurred in problems where the method of solution was not apparent, but was dependent upon deductions based on experiences with the idea . . .
6. The real reasons for missing most of the problems were not computational ones (p. 203).

In 1967, Gorman examined 293 studies on problem solving and rejected all but 37 of them on the basis of poor validity. The main point in his report was that even in the 37 accepted studies, attention had not been adequately given to the thinking process during the solution of verbal problems. He felt that there was a strong need to develop some method of monitoring the process of thinking during problem solving.

Kilpatrick (1967) had been working on this very problem at the time Gorman wrote his dissertation.

> To make our knowledge of problem solving relevant to education, we must eventually study how students solve problems of the sort they meet in the classroom. If at present there are too many uncontrollable and even unknown sources of variation for careful experimental studies, we should at least attempt analyses of behavior. Such analyses, though they run a high risk of producing little immediate payoff, are necessary for directing future experimentation (Kilpatrick, 1967, p. 2).

He felt that the "thinking-aloud" technique was the best available technique for tracing behavior of an individual engaged in problem solving. Thinking aloud, if tape recorded in conjunction with any written notes by the observer allows lengthy observations of any processes involved in problem solving.

There are, however, disadvantages in using this method, which Kilpatrick recognized. Thinking aloud may tend to inhibit speech. Some subjects remain silent during the most productive periods of thought. This forces the observer to either make inferences about the nature of this nonverbal behavior, or to ask the child to explain what sequence of thought he had performed. The first alternative is highly undesirable if the subject has nonverbally completed more than one step. The latter in some

cases may interrupt a chain of thought which inhibits the subject's performance. In addition, a subject may go about solving a problem differently when asked to vocalize his thoughts. Kilpatrick was convinced that it is better to risk these objections than to not be able to analyse problem-solving processes, as most of them are performed in the mind and nothing is written down by the subject.

Several studies have attempted to analyse the effect of verbalization on problem solving. Hafner (1957), Gagne and Smith (1962), and Roth (1966) found no significant differences on mean scores, solution time, or modes of inquiry between pupils required to verbalize as they solved problems and those who worked silently.

Kilpatrick concluded, after analysing his tape recorded interviews, that the trial and error method was widely used by both boys and girls, and suggested that teachers teach the intelligent use of this technique. The impulsive child was found to implement the first idea that comes to him. When he fails to solve the problem in this way, his increased anxiety may eventually lead to a pattern of apathy or hostility toward the problem situations.

III. INDIVIDUALIZED INSTRUCTION

Keuscher (1970) suggests that individualization of instruction involves four vital dimensions. He maintains that provision must be made for each of the four if one is to have a truly individualized program.

> It is not enough simply to diagnose differences among students and to prescribe a variety of learning activities, important as custom-tailored programs for individuals may be in their own right. Nor is it enough to tinker with the school's curriculum organizational pattern, although adjustments in both may be appropriate in any comprehensive plan to take care of individual differences.
>
> Likewise, individualization is more than allowing students to make decisions and to assume responsibility for their own educational programs, despite the fact that democracy can only thrive where

> citizens have learned to be independent, self-directed, and responsible.
> And, individualization of instruction is more than encouraging and nurturing uniqueness and creativity, although the very survival of our society may depend on pursuit of these qualities (Keuscher, 1970, p. 16-17).

He claims that an individualized program must be characterized by more teacher awareness of individual differences and provision for more alternative learning activities. Both curriculum and organizational patterns must be flexible enough to allow for modification when pupil needs dictate it. Students should be encouraged to make decisions about what, when and how they study.

Individually Prescribed Instruction

IPI was jointly developed by Glaser, Bolvin and Lindvall, at the University of Pittsburgh. The project and procedures known as IPI were first used in September, 1964, in the Oakleaf Elementary School of the Baldwin-Whitehall School District, in Pittsborgh, Pennsylvania (Lindvall and Bolvin, 1967). Three basic assumptions were made during or previous to the development of IPI:

> 1. Learning is something that is ultimately personal and individual. .....teaching machines and programed textbooks have exemplified a useful approach to the individualization of instruction. They have shown that if conditions are arranged so that a pupil can work through a sequence of learning experiences that are carefully graded in terms of increasing difficulty, each pupil can progress at his own individual pace and with little or no outside assistance to acquire relatively complex skills and types of knowledge.
> 2. A second assumption of the IPI project is that the same type of planning and "programing" that is employed in a programmed textbook can be used to develop a more extensive program which extends over grade lines, covers at least all of the elementary-school years, and involves a much greater variety of types of learning experiences than can be presented in a textbook....
> 3. Still another assumption basic to the planning of the IPI procedure was the idea that if principles of programming were applied, the desired flexibility in rates of pupil progress would not involve any plan for "special promotions," for "retentions," or any type of complex grouping or regrouping (Lindvall and Bolvin, 1967, p. 233-4).

The IPI program was then developed. The primary objective, obviously, was to accomodate each individual's learning needs and to take into account individual learning characteristics. More specifically, the objectives were:

1. To enable each pupil to work at his own rate through units of study in a learning sequence.
2. To develop in each pupil a demonstrable degree of mastery.
3. To develop self-initiation and self-direction of learning.
4. To foster the development of problem solving thought processes.
5. To encourage self-evaluation and motivation for learning (Scanlon and Bolvin, undated, p. 5).

Research that has been done involving IPI schools has been carried out primarily by graduate students from the University of Pittsburgh. Yeager (1966) reported that rate of learning is specific to a given task for a particular person. He found that intelligence was a poor predictor of the rate at which individuals progressed through a given unit of work. A subsequent study supported Yeager's findings (Wang, 1968).

There are at least two ways of individualizing a particular program. The first way is to allow students to work at their own rate through the same course of studies. The second way is to modify the course of studies and present gifted students with concepts treated in a more rigorous manner than those presented to less able students. IPI has individualized in terms of the first method. "The individualization that has been achieved so far is largely in terms of rate of pupils progress (Lindvall and Bolvin, 1967, p. 252)."

With respect to achievement, most studies have shown that students achieve as well in IPI schools as in control schools (Fisher, 1967), and in addition, they are more favorably disposed to mathematics (Research for Better Schools, 1969). Fisher found that students in IPI schools did as well as two other methods of teaching on <u>Metropolitan Achievement Tests</u>.

Fisher found no significant difference between the IPI group and the control group with respect to problem-solving ability. He points out that these tests favor graded schools because they test at any grade level those topics that are typically taught at the particular grade level. Students in IPI classes may have spent most of the school year learning concepts associated with previous grade levels which have not yet been mastered.

Lipson (1967) summarized the results of tests done in the Baldwin-Whitehall School District during three years that IPI had been in use. He reported that after the first year of operation, almost all first and second grade students ranked above the 80th percentile. The third and fourth grades appeared average, while the fifth and sixth grades has large numbers of students ranking below the 40th percentile. The explanation given for the poor showing at the fifth and sixth grades was that these students had not covered much of the material normally presented to students in the upper grades. Many students spent most of the year going below their grade level to overcome deficiencies in lower grade material, hence they were not learning the material contained in the standardized tests on which they were evaluated.

Junior high school teachers reported that these same grade six students appeared no different from other students in the school during the grade seven year. They performed equally well even though they had spent the previous year mastering earlier objectives and not encouraging certain topics.

In a study examining the effect of IPI on students of different ability levels, Deep (1966) found that although students of average I.Q. seemed to gain numerically more being in a IPI school, statistically the

study showed that the IPI program did not operate differentially among high, average and low ability students. Average students were those student ranging in I.Q. scores from 90 to 110. He drew his conclusions on the basis of results of standardized tests in arithmetic concepts and problem solving.

Other Individualized Programs

Bartel (1965) described a study designed to test the feasibility of building a mathematics curriculum which would include both the "new" mathematics and a program of individualized instruction, at the grade four level. The grade four course was divided into eight strands or topics. Worksheets were constructed containing references to textbooks which were available to the students. Each student selected his own strand of of mathematics to study. He did all computations, corrections and scoring of worksheets, which he kept in a folder for teacher-pupil conferences.

The general consensus of the teachers involved was that in addition to allowing students to move at their own rate, children in the indiviualized program developed more independence, self-confidence, responsibility in checking their own work, and were happier and more relaxed than students in the control group.

Bartel came to three conclusions. She found no significant difference in achievement scores, as measured by the Iowa Tests of Basic Skills, between the individualized group and the control group. She found that neither group favored students of low, average, or above average I.Q. Finally, no significant differences between the achievement of boys and girls in either group was found.

Three years later a study was carried out at the University of Houston to determine the effect of individualized problem-solving assign-

ments on the achievement of fifth grade pupils in mathematics concepts and problem solving (Nabors, 1968). The assignments were individualized in terms of the pupil's reading ability but contained problems of varying difficulty. Three times a week for ten weeks the experimental group worked on the individualized assignments while the control group used only assignments from the regular textbook.

Nabors also used the Iowa Tests of Basic Skills to measure achievement. He compared students of low, average, and above-average intelligence and reading abilities. No differences between the control group and the experimental group were found on the arithmetic concepts subtest. The only difference he found between the experimental and the control group on the Problem-Solving Subtest was between students of average intelligence. Students of average intelligence in the experimental group gained significantly more than students of average intelligence in the control group.

Another individualized program being developed, called a Program for Learning in Accordance with Needs (PLAN) is described as being exceeded only by IPI in terms of the number of students involved (Phi Delta Kappan, 1970). Whereas IPI has developed its own curriculum, PLAN utilizes available commercial materials, adjusting them to the individualized program. PLAN is the only individualized program involving language arts, reading, social studies, mathematics, and science in grades one to twelve.

PLAN uses a guidance program and computers to help teachers by monitoring pupil progress, assigning learning tasks, suggesting ways to accomplish these tasks and testing students. In this way a student is assisted in developing an ability to manage a program he has partially planned for himself.

Results are not yet available, but a thorough evaluation is being

planned at this time. The per student cost is estimated at approximately one hundred dollars per year.

The emphasis which is being placed on individualization of instruction is not unique to North America. In Sweden, the National Board of Education started an Individualized Mathematics Instruction (IMU) program in the fall of 1963. The completion date for this program has been set for 1971. The aim of the project was to construct materials in mathematics with which a student could instruct himself (Oreberg, 1968). Students write a diagnostic test for placement purposes, and then he is given a module of work to complete. On completion, he is given a prognostic test. Each module takes approximately one month to complete.

Although formative evaluation has been continuous, a full scale evaluation report of the project is expected at the completion of the 1970-71 year. The IMU group will be compared to a control group. In addition an evaluation will be carried out to determine the optimum teacher-student and teacher assistant-student ratios necessary to utilize the program.

IV. SUMMARY

The first two sections of this chapter were devoted to a discussion of the verbal problem of arithmetic and the processes which an individual uses when solving problems. Several important points seem worth summarizing.

A verbal problem exists when an individual is attempting to solve a quantitative situation which involves the use of some process which is not immediately obvious or automatic for that individual. In addition, some educators believe that to be a verbal problem, there must also be some application of mathematics to a life situation.

In terms of the stages or phases of the problem-solving process, there is not quite unanimous agreement among educators. It is at least safe to say that if an individual does not become involved, a problem does not exist, hence there will be no phases used. When solving a problem, assuming acceptance of the problem, an individual at least will define the problem in terms of the relationships of the variables involved, he will outline some plan of attack, and finally he checks to see if his plan was useful in finding a solution. In other words, he <u>senses</u> the situation, he <u>predicts</u> various hypotheses which can be used to solve the problem, and he <u>verifies</u> the feasibility of these hypotheses.

The final section of the chapter contained a discussion of individualized instruction. Included was a description of the IPI mathematics program and some related research. The IPI program appears to have achieved individualization in terms of rate of learning. On the basis of marks on standardized tests, students using an IPI program as curriculum have achieved very similar results as students in conventional classes.

CHAPTER THREE

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

This chapter describes the population and the sample of the study, a description of the arithmetic programs in the schools, a description of the tests used including the procedure of administration, and a discussion of the statistical procedures used to analyse the data.

I. POPULATION AND SAMPLE

The population for the study consisted of all 266 grade six pupils enrolled in the six schools involved in the IPI field study. This included all grade six pupils in Forest Heights School in Edmonton, St. Vincent De Paul School in Calgary, Millarville School in the Foothills School Division, Princeton School in Edmonton, St. Leo School in Calgary, and Red Deer Lake School in the Foothills School Division. The first three schools named are IPI schools and the latter three are their respective controls.

The reason that grade six was selected for evaluation was two-fold. First a study was done in 1970 by Affolter in which an oral interview technique was used to identify strategies used by grade six children while solving verbal problems of arithmetic. It seemed valuable to determine whether the test developed by Affolter could be used as an evaluation instrument when two programs are being compared. Secondly, Lazerte (1933) claimed that:

> Until the age of ten or eleven years rather unitary habits and concepts are suitable. An integration of the basic constituent abilities appears to take place at about the grade V level. The

> more systematic and thorough the groundwork, the earlier the integration takes place (p. 124-5).

On this basis, it was believed that if differences, in strategies used or frequency of strategies used do exist amongst children in IPI schools and those in conventional schools, those differences will be greater at the grade six level than at any previous grade level.

The sample consisted of thirteen students from each of the six schools involved. A table of random numbers was used to select the thirteen students (Keeping, 1962). All students in the sample had been enrolled in the same school for the past two years. Nine students were excluded from the population when selecting the sample because they had been absent from school when the Arithmetic Problem-Solving Subtest of the Canadian Tests of Basic Skills was administered. The sample then consisted of 39 students in the IPI group and 39 students in the control group. No attempt was made to stratify on the basis of sex, however, all but one of the groups of thirteen were as evenly divided as possible. Table I gives the sex distribution in each of the samples and in each of the school populations. Sample and population mean I.Q. scores as measured by the Canadian Lorge-Thorndike Tests of Intelligence are summarized in Table II. Age and I.Q. distributions for the sample are summarized in Tables III and IV respectively.

In an investigation of teacher's perceptions of IPI Connors (1970) identified St. Vincent De Paul as representing a higher socio-economic area, Forest Heights as representing a lower socio-economic area, and Millarville as representing a rural area. Connors used school records to ascertain the occupation of the family head who was the main source of income for the family. St. Vincent De Paul was given a rating of 56.63 on the "Blishen Socio-Economic Index," while Forest Heights was

TABLE I

SEX DISTRIBUTION IN SAMPLES AND POPULATIONS

School	Boys		Girls	
	Sample	Population	Sample	Population
St. Vincent De Paul	6	23	7	25
Millarville	7	9	6	9
Forest Heights	6	30	7	26
St. Leo	7	18	6	21
Red Deer Lake	5	13	8	15
Princeton	7	33	6	36

TABLE II

MEAN I.Q. SCORES OF SAMPLE AND POPULATION

School	Verbal		Non-Verbal	
	Sample	Population	Sample	Population
St. Vincent De Paul	105.46	109.88	115.61	113.54
Millarville	102.69	103.86	106.31	108.21
Forest Heights	104.23	100.96	99.62	101.37
St. Leo	109.15	110.87	114.15	112.59
Red Deer Lake	103.23	100.50	111.77	109.28
Princeton	98.38	103.99	105.38	108.27

given a rating of 42.69. Blishen socio-economic index scores range from 76.69 to 25.36, with a mean index of 42.15. Forest Heights then does not represent a low socio-economic area, but it does represent a lower socio-economic area than does St. Vincent De Paul.

The IPI classroom is characterized by pupils working independently, proceeding on their own to various places in the classroom or school, and

TABLE III

AGE DISTRIBUTION OF SAMPLE

Interval in Months	IPI Group			Control Group		
	St. Vin.	Mill.	F.H.	St. Leo	R.D.L.	Prince.
131-135	2			1		
136-140	5	4	4	2	2	4
141-145	5	6	8	6	7	2
146-150	1	2		2	3	4
151-155		1		1		1
156-160			1			2
161-165					1	
166-170						
171-175				1		
Total	13	13	13	13	13	13

by a number of teacher-aides correcting completed assignments and exams. Each unit of work is usually one to eight pages in length.

When a child enters an IPI mathematics program he is given a <u>placement test</u> to find out what abilities he has in areas such as numeration, measurement, addition, subtraction, etc. On this basis, he is given a

prescription, which lists the objectives along with the material that the pupil will begin studying. Any classroom is adequate for seating students who are working on their prescribed materials.

TABLE IV

I.Q. DISTRIBUTION OF SAMPLE

	IPI Group		Control Group	
Interval	Verbal	Non-Verbal	Verbal	Non-Verbal
Below 80	1	1	2	1
80-89	5	3	6	3
90-99	8	5	8	6
100-109	14	13	10	9
110-119	6	6	7	8
120-129	3	10	5	6
130-139	2	1		5
140-149			1	1
Total	39	39	39	39

For example, a class of 60 or 75 pupils would probably use at least a double-sized classroom. Two or three teachers would provide instructional assistance, while two to four clerks would distribute materials and grade papers. On completion of the prescribed materials each pupil writes a curriculum embedded test, which plays a part in determining what a pupil does next. When the clerk has corrected this test, if 85% mastery has been obtained, the pupil is given another prescription assigning different work, but in the same area and same level as before.

Also included in the sequence through which a student works in

any given area are periodic unit tests. These unit tests, which do not occur as frequently as the curriculum embedded tests, are given before and after sizeable units of work have been covered. Their use is primarily for making decisions relative to pupil progress (Lindvall and Bolvin, 1966).

All students in the control group were using the textbook Seeing Through Arithmetic Book 6 as the basis for their arithmetic program. Red Deer Lake School had only one grade six class. St. Leo and Princeton schools each had two grade six classes and in both cases they were situated in an open area classroom with two teachers responsible for the entire grade six population. In all three cases, although the textbook was followed very closely, supplementary exercises were provided through mimeographed worksheets.

II. PILOT STUDY

A pilot study was conducted in two Edmonton elementary schools in order to acquaint the investigator with the testing procedures used by Affolter (1970) and to refine the Strategies of Problem Solving Test. Eighteen students were involved in the pilot study, which was carried out in February, 1971. The Strategies Test was orally administered to each of the eighteen grade six students. It was found that very few of th eighteen students could explain what process they were using when they were actively engaged in problem solving. They preferred to work through a question first and then explain what they had done.

III. INSTRUMENTATION

Prior to the implementation of the IPI program, in September, 1969, the Canadian Lorge-Thorndike Tests of Intelligence and the Canadian Tests

of Basic Skills were administered to all students in all six schools. The I.Q. scores and the scores on the Problem-Solving Subtest of the Canadian Tests of Basic Skills, that were gathered in 1969, were collected and used in this study.

In addition, the Problem-Solving Subtest of the Canadian Tests of Basic Skills was administered to the total grade six population of the six schools involved on April 6 and April 7, 1971. The tests were administered exactly as described in the teacher's manual, including the 30 minute time limit.

A major part of this study was to measure and compare the relative efficiency of the strategies used by grade six pupils in IPI classes with the strategies used by grade six pupils in conventional elementary schools, when solving verbal problems of arithmetic. This necessarily implies that a test was required which consisted of items, the solution of which was not immediately obvious or automatic. For this reason, the Strategies of Problem Solving Test constructed by Affolter (1970) was adopted.

The Strategies Test consisted of seven mathematical problems which were not the type usually encountered in elementary textbooks. The test and criteria for scoring are included in Appendix A. The test was given as an oral examination in an attempt to reduce the influence of reading ability as well as to analyse the thinking processes of the subjects while engaged in problem solving.

It was decided, by the investigator, to replace two of the seven test items. Test item II was replaced because it did not fit the investigator's definition of a verbal problem of arithmetic because there was no arithmetic operation required to solve the problem. Test item IV was also replaced because it seemed to be too difficult for grade six children.

The difficulty index of item IV in the study by Affolter (1970) was .40. In addition, the investigator found that none of the first ten students in the pilot study were able to solve item IV.

Before the testing began the subjects were told that the investigator was giving the test in an attempt to find out how grade six students solve problems and what kinds of problems interest them. They were asked to think aloud while they solved each problem. It was made clear that if they could not think aloud while they solved a problem, they could solve the problem and then explain what steps they they had taken to solve the problem.

At the same time each subject was told what questions the investigator would ask if the subject had not adequately explained his method (See Appendix A). He was told that the questions would be asked whether his method was correct or not and that the questions would not imply correctness or incorrectness of his method or solution. Finally, he was told that he would be given computational assistance if requested.

The oral test was administered to thirteen students in St. Vincent De Paul and thirteen students in St. Leo schools during the week of April 19-23, 1971, and to thirteen students in Millarville and thirteen students in Red Deer Lake schools during the week of April 26-30, 1971, and to thirteen students in Forest Heights and thirteen students in Princeton schools during the week of May 3-7, 1971. The testing took approximately two days for each school in the study. Tape recordings of the test interviews were transcribed by the investigator during the evening of the same day that they were recorded.

The transcriptions of the oral test were scored after all the interview had been completed. In addition to the transcriptions, rough

work used by the subjects was available to assist the marker. The first question on the oral test was designed as a practice question, consequently no marks were awarded for it. The subjects were made aware of the status of question one before the test began. Questions two to seven were marked out of a possible score of six marks for each question. The criteria used for scoring each question is included in Appendix A.

Marker reliability was checked by having two competent graduate students in mathematics score ten randomly selected tests. The scoring of the investigator and the other two judges was compared using a one-way analysis of variance with repeated measures. The reliability indexes derived from this analysis were .96, .85, .91, .93, .89, and .98 for items II, III, IV, V, VI, and VII respectively. The reliability index for the total test score was .97. Table V summarizes the scoring of the ten tests.

IV. DATA ANALYSIS

A correlation matrix whose elements were Pearson product-moment correlations between variables was obtained to test hypothesis 3. The covariates used when testing hypotheses 1 and 2 were selected on the basis of the correlations obtained when testing hypothesis 3.

Multiple linear regression analysis (MULR05) was used to test hypotheses 1 and 2. Non-verbal I.Q. scores and achievement scores on the Problem-Solving Subtest of the Canadian Tests of Basic Skills, administered in September, 1969, were used as covariates when testing hypotheses 1 and 2.

Two-way analysis of variance (ANOV25) was chosen as the statistical means of analysing hypothesis 4. Interaction was tested with respect to three intelligence levels and scores obtained from the Canadian Tests

TABLE V

COMPARISON OF TEST SCORES FOR TEN SUBJECTS WITH THREE JUDGES

Subject Number	Problem:	II			III			IV			V			VI			VII			Test Total		
	Judge:	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
125		6	6	6	0	0	1	3	3	3	3	3	4	5	5	3	0	0	0	17	17	17
233		4	4	3	1	1	1	3	3	3	6	6	6	6	6	6	6	6	6	26	26	25
239		6	6	6	1	1	1	3	3	3	6	6	6	3	3	2	4	3	3	23	22	21
304		6	6	6	1	1	1	6	6	6	5	5	5	0	3	3	0	0	0	18	21	21
408		0	2	1	0	0	0	4	2	3	4	3	3	6	6	6	2	2	1	16	15	14
416		6	6	6	4	4	3	3	3	3	3	3	4	6	6	6	0	0	0	22	22	22
505		2	1	1	4	3	2	4	2	3	3	2	3	3	3	3	0	1	1	16	12	13
515		6	6	6	0	1	1	3	1	2	4	4	5	0	0	1	0	1	0	13	14	15
547		6	6	6	6	6	6	3	3	3	6	6	6	6	6	6	0	0	0	27	27	27
643		6	6	6	1	1	1	6	6	6	6	6	6	6	6	6	6	6	6	31	31	31

<u>of Basic Skills</u> and the Strategies Test.

Using a one-way analysis of variance, the IPI sample was compared to the control sample with respect to I.Q. scores and age. The results are summarized in Table VI.

TABLE VI

ANALYSIS OF VARIANCE BETWEEN IPI AND CONTROL SAMPLES

Variables	IPI		Control		F	P
	Mean	SD	Mean	SD		
Verbal I.Q.	104.13	14.02	103.59	15.66	.03	.872
Non-verbal I.Q.	107.18	13.97	110.44	15.92	.92	.340
Age(months)[a]	141.82	4.78	145.38	7.33	6.46	.013

[a]Homogenity of of variance test: CHISQ=6.64, p=.010

CHAPTER FOUR

THE FINDINGS OF THE STUDY

The findings of the study are presented in two sections. The first section contains the results of analysis done with respect to the hypotheses presented in Chapter One. The second section describes additional findings not specifically included in the major hypotheses of the study.

I. THE FINDINGS WITH RESPECT TO THE HYPOTHESES

Hypothesis 1: There is no significant difference between the IPI groups and the control groups with respect to:

(a) problem-solving ability as measured by an oral test

(b) verbal problem-solving achievement as measured by a written test.

The hypothesis was tested with a one-way analysis of covariance technique using a multiple linear regression approach. This method involves constructing two models and then comparing their ability to predict the criterion. The first model contains information about group membership for each subject and the covariate(s). The second model only contains information about the covariate(s). Any difference in the ability of these two models to predict the criterion is the result of differences in group membership, because knowledge of group membership is the only difference between the two models.

The testing of hypothesis 1(a) involved comparing the IPI sample with the control sample on the basis of test scores obtained on the Strategies

Test. The CTBS scores obtained by the same two groups were compared when testing hypothesis 1(b). The summary of analysis of variance of these two test scores is presented in Table VII.

Hypothesis 1(a) was tested twice using different combinations of covariates. In the first instance the only covariate used was non-verbal I.Q. and scores obtained on the CTBS which was administered in September, 1969 were used as covariates. As a result probabilities of .627, and .473 were obtained. Table VIII summarizes the results of the analysis. On this basis hypothesis 1(a) was accepted. Although the control group achieved slightly higher results, the difference was far from being significant.

When testing hypothesis 1(b) non-verbal I.Q. scores and the CTBS scores obtained in September, 1969 were used as covariates. The analysis revealed an F ratio of 13.598. The probability of obtaining an F ratio of this size was calculated to be .0004. A more complete summary of the results of the analysis is outlined in Table IX.

The control group achieved significantly higher than the IPI group on the CTBS in terms of unadjusted mean scores. When controlling for I.Q. and initial differences in problem-solving achievement (CTBS, 1969), the achievement of the control group remained significantly higher than the IPI group in terms of problem-solving achievement. Hypothesis 1(b) must therefore be rejected.

Hypothesis 2: There is no significant difference among the three IPI schools with respect to:

(a) problem-solving ability

(b) verbal problem-solving achievement.

Hypothesis 2 was also tested with a one-way analysis of covariance technique using a multiple linear regression approach.

TABLE VII

ANALYSIS OF VARIANCE BETWEEN IPI AND CONTROL GROUPS

Variables	IPI		Control		F	P
	Mean	SD	Mean	SD		
Strategies Test [a]	17177	6.84	17.92	7.35	.01	.92
CTBS, 1971[b]	13.85	5.48	19.36	5.58	19.36	.00004

[a]Maximum score- 36 [b]Maximum score- 31

TABLE VIII

ONE-WAY ANALYSIS OF COVARIANCE OF HYPOTHESIS 1(a)

Covariate(s)	DF		R_1^2	R_1^2	F	P
	Num.	Den.				
Non-verbal I.Q.	1	75	.285	.283	.237	.627
Non-verbal I.Q: and CTBS, 1969	1	74	.290	.285	.520	.473

TABLE IX

ONE-WAY ANALYSIS OF COVARIANCE OF HYPOTHESIS 1(b)

Covariates	DF		R_1^2	R_1^2	F	P
	Num.	Den.				
Non-verbal I.Q. AND CTBS, 1969	1	74	.536	.450	13.598	.0004

The testing of hypothesis 2(a) involved comparing the three IPI schools on the basis of test scores obtained on the Strategies Test. The CTBS scores obtained by the three schools were compared when testing hypothesis 2(b). Table X outlines the unadjusted mean scores obtained by the three IPI schools on the above tests.

TABLE X

MEAN SCORES OBTAINED BY THREE IPI SCHOOLS ON THE STRATEGIES TEST AND THE CTBS, 1971

School	Strategies Test		CTBS, 1971	
	Mean	S.D.	Mean	S.D.
St. Vincent De Paul	19.85	8.54	13.54	5.83
Millarville	17.23	6.58	15.69	6.06
Forest Heights	16.23	4.94	12.31	4.23

When testing hypothesis 2(a) each school was compared to each of the other two schools. They were compared using non-verbal I.Q. as the covariate. The probabilities obtained on the three analyses ranged from .409 to .981. Table XI presents a more complete summary of the analyses. On the basis of these analyses no difference between any of the IPI schools could be identified. Hypothesis 2(a) was therefore accepted.

Hypothesis 2(b) was tested using non-verbal I.Q. scores and the CTBS scores gathered in September, 1969 as covariates. Again, each school was compared to each of the other schools. The probabilities obtained ranged from .036 to .333. Table XII summarizes the analysis. Since no

TABLE XI

ANALYSIS OF COVARIANCE FOR THE VARIABLE OF PROBLEM-SOLVING ABILITY[a] AMONG THE THREE IPI SCHOOLS

Source	Covariate(s)	DF(num.)		R_1^2	R_1^2	F	P
		Num.	Den.				
Difference between St. Vincent De Paul and Millarville	Non-verbal I.Q.	1	23	.250	.250	.004	.950
Difference between St. Vincent De Paul and Forest Heights	Non-verbal I.Q.	1	23	.077	.049	.706	.409
Difference between Millarville and Forest Heights	Non-verbal I.Q.	1	23	.125	.125	.001	.981

[a]Measured by the Strategies Test

TABLE XII

ANALYSIS OF COVARIANCE FOR THE VARIABLE OF PROBLEM-SOLVING ACHIEVEMENT[a] AMONG THE THREE IPI SCHOOLS

Source	Covariate(s)	DF		R_1^2	R_2^2	F	P
		Num.	Den.				
Difference between St. Vincent De Paul and Millarville	Non-verbal I.Q. and CTBS, 1969	1	22	.352	.204	4.998	.036
Difference between St. Vincent De Paul and Forest Heights	Non-verbal I.Q. and CTBS, 1969	1	22	.294	.263	.981	.333
Difference between Millarville and Forest Heights	Non-verbal I.Q. and CTBS, 1969	1	22	.343	.297	1.558	.225

[a]Measured by the CTBS, 1971

differences between the three IPI schools could be identified, hypothesis 2(b) was accepted at the .01 level of significance. Hypothesis 3: There is no significant relationship between problem-solving ability as measured by an oral test and:

(a) verbal problem-solving achievement

(b) intelligence

(c) age

Hypothesis 3 was tested by examining a correlation matrix which was created by forming Pearson product-moment correlations among the following variables: problem-solving ability scores, verbal problem solving achievement scores gathered in 1969 and 1971, verbal I.Q. scores, non-verbal I.Q. scores, and age. The matrix obtained is provided in Table XIII.

The Pearson product-moment correlation is an expression of the relationship between two variables. An index of +1.00 indicates a perfect direct relationship. When a perfect inverse relationship occurs, the index becomes -1.00. An index of 0.00 indicates that no linear relationship exists. A Pearson product-moment correlation can be used meaningfully only when the two variables being related are continuous and are linearly related.

Since a significant positive relationship exists between scores on the Strategies Test and scores on the CTBS, hypothesis 3(a) was rejected. Hypothesis 3(b) was also rejected because of the direct relationship between scores on the Strategies Test and both intelligence scores. From Table XIII we see that age is only slightly negatively correlated to the other variables, therefore, hypothesis 3(c) must be rejected. Hypothesis 4: There is no significant interaction between pupil intelligence and type of school with respect to:

TABLE XIII

INTERCORRELATIONS AMONG MAJOR VARIABLES

Variable	1	2	3	4	5	6
1. Verbal I.Q.	1.000	0.710**	-0.440**	0.467**	0.486**	0.365**
2. Non-verbal I.Q.		1.000	-0.364**	0.584**	0.592**	0.525**
3. Age			1.000	-0.042	-0.011	-0.070
4. CTBS,1969				1.000	0.590**	0.345**
5. CTBS, 1971					1.000	0.418**
6. Strategies Test						1.000

**Significant at the .01 level

(a) problem-solving ability

(b) verbal problem-solving achievement.

Three I.Q. levels, based on non-verbal I.Q. scores, were established. In the control group 10 subjects were categorized as having a low I.Q., 17 subjects were categorized as having an average I.Q., and 12 subjects were categorized as having a high I.Q. In the IPI group 9, 19, and 11 subjects were identified as having low, average and high I.Q.'s respectively.

Hypothesis 4 was tested using a two-way analysis of variance to determine whether subjects in any I.Q. level benefit from the type of school in which they are enrolled.

A two-way analysis of variance first performs a test for additivity to test for any interaction effect. Main effects are first tested under the assumption that no interaction exists, and then tested under the assumption that the interaction is significant. Cell means and variance are also calculated.

Below average, average, and above average groups in the control schools achieved mean scores of 11.40, 18.12, and 23.01 respectively on the Strategies Test measuring problem-solving ability. In the same order, the three groups in the IPI schools acheived mean scores of 14.44, 18.16, and 19.82 on the same test. On the test for additivity, summarized in Table XIV, an F ratio of that size was calculated to be .285, which indicated there was no significant interaction. Hypothesis 4(a) was accepted on this basis.

With respect to the CTBS which was administered in April, 1971 the three ability groups in the control schools achieved mean scores of 14.10, 18.64, and 24.75, while the three ability groups in the IPI schools

had mean scores of 10.78, 13.63, and 16.73. In the analysis of hypothesis 4(b) the test for additivity, summarized in Table XV, revealed an F ratio of 1.451. The probability subsequently calculated was .241, which indicated no significant interaction. Hypothesis 4(b) then, was also accepted.

II. ADDITIONAL RESULTS FROM THE ANALYSIS OF DATA

The testing period and the subsequent marking of the tape recorded interview provided points of interest not covered directly by the major hypotheses.

Affolter (1970) found that the more successful problem solvers exhibited more "tentative thinking" than poor problem solvers. Although no formal analysis was performed in this study, it was very obvious that this premise was also true in this study. There seemed to be no difference in the number of "tentative thinkers" in either the IPI or control samples.

When a subject encountered a problem which he did not recognize as being familiar, quite often he resorted to a trial-and-error as a method of solution most frequently when solving Problem III. The investigator found that most students were so confident that the ratio method was the correct method to use, that trial-and-error was seldom used when attempting Problem III.

Subjects involved in the process of problem solving were very hesitant to ask for computational assistance even though the researcher had explicitly stated that help would be given if requested, and that such assistance would not affect the mark awarded for the problem. However, computation was a source of error, the basic concept of division usually was not understood.

The main source of error in Problems III and VII was in the process

TABLE XIV

TEST FOR ADDITIVITY ON THE CRITERION OF PROBLEM-SOLVING ABILITY[a]

Source	SS	DF	MS	F	P
Interaction between type of school and ability levels	10.43	2	52.15	1.278	.285
Within	2937.47	72	40.80		

[a]Measured by the Strategies Test

TABLE XV

TEST FOR ADDITIVITY ON THE CRITERION OF PROBLEM-SOLVING ACHIEVEMENT[b]

Source	SS	DF	MS	F	P
Interaction between type of school and ability levels	61.09	2	30.54	1.451	.241
Within	1515.20	72	21.04		

[b]Measured by the CTBS, 1971

of sensing what was given and what was required in a given problem. In the remaining problems, the main source of error was the failure to predict a reasonable method of solution.

Although most subjects were not able to think aloud while engaged in the process of problem solving, virtually all students were able to satisfactorily explain the process they had used in solving the problem. Very frequently, as a subject was explaining his method of solution, errors were discovered and corrected. During the process of verbalizing subjects seemed to gain one extra opportunity to think through both what is given and what is required in each problem.

The mean age score for the IPI group was 141.82 months, while the mean age score for the control group was 145.38 months. The variance for the IPI group was 22.85 as opposed to 53.68 for the control group. Age was tried as a covariate in analysis of covariance on the criteria of problem-solving ability and problem-solving achievement. No significant results were recorded.

An analysis of variance was performed to compare the scores of boys in the sample with the scores of girls in the sample. The girls in the sample had higher I.Q. scores, and at the same time were older. These results are not surprising due to the fact that the older children in any particular grade are more likely to have been held back a year in school and therefore are more likely to have lower I.Q scores. Table XVI summarizes the comparison of boys and girls, though not significantly, on the Strategies Test.

The most interesting extra information appeared when the analysis used to test hypothesis 3 was performed on the IPI group and control group individually. Tables XVII and XVIII summarize the analyses. All correlations that were significant when the total sample was tested

remained significant when the extra analysis was performed on the control scores. However when the extra analysis was performed on the IPI scores verbal I.Q. did not correlate significantly with the Strategies Test, non-verbal I.Q. did not correlate significantly with age and the CTBS scores gathered in 1969 did not correlate significantly with the 1971 CTBS scores or the scores on the Strategies Test. Two correlations, previously significant at the .01 level were reduced to the .05 level. These were non-verbal I.Q. with the Strategies Test and the CTBS, 1971 with the Strategies Test. Two reasons may explain part of the findings. First, students in the IPI group seemed give more random responses than students in the control group with respect to ability. Secondly, 11 students in the IPI group scored less on the 1971 CTBS than on the 1969 CTBS as opposed to only two students in the control group.

The average two year gain in CTBS scores for the control group, calculated by subtracting the 1969 CTBS score from the 1971 CTBS score, was 5.5. The average gains for the IPI schools were 1.8 for St. Vincent De Paul, 6.5 for Millarville, and 3.2 for Forest Heights. St. Vincent De Paul had a higher average non-verbal I.Q. score than any of the other five schools in the study.

The results of the Strategies Test showed all six schools in the same rank order as the results of the non-verbal intelligence test. St. Vincent De Paul scored highest on the Strategies Test while Forest Heights, the school with the lowest average non-verbal I.Q. score, scored lowest on the Strategies Test.

III. SUMMARY OF RESULTS

The principal findings in relation to the major hypotheses are briefly summarized:

TABLE XVI

A ONE-WAY ANALYSIS OF VARIANCE: BOYS VS GIRLS

Variable	Boys		Girls		F	P
	Mean	SD	Mean	SD		
Verbal I.Q.	99.53	14.74	107.98	13.74	6.86	.01
Non-verbal I.Q.	104.32	14.76	113.08	14.05	7.21	.01
Age (months)	145.13	8.03	142.15	3.91	4.41	.04
CTBS, 1969	10.95	4.05	12.50	4.89	2.32	.13
CTBS, 1971	15.55	5.95	17.60	6.26	2.19	.14
Strategies Test	18.92	6.95	16.83	7.09	1.74	.19

TABLE XVII

INTERCORRELATIONS AMONG MAJOR VARIABLES: IPI

Variable	1	2	3	4	5	6
1. Verbal I.Q.	1.000	0.639**	-0.459**	0.411**	0.465**	0.144
2. Non-verbal I.Q.		1.000	-0.282	0.512**	0.457**	0.355*
3. Age			1.000	-0.111	0.020	0.090
4. CTBS, 1969				1.000	0.310	0.133
5. CTBS, 1971					1.000	0.378*
6. Strategies Test						1.000

**Significant at the .01 level

*Significant at the .05 level

TABLE XVIII

INTERCORRELATIONS AMONG MAJOR VARIABLES: CONTROL

Variables	1	2	3	4	5	6
1. Verbal I.Q.	1.000	0.761**	-0.448**	0.573**	0.619**	0.541**
2. Non-verbal I.Q.		1.000	-0.486**	0.632**	0.732**	0.656**
3. Age			1.000	-0.191	-0.273	-0.177
4. CTBS, 1969				1.000	0.654**	0.531**
5. CTBS, 1971					1.000	0.528**
6. Strategies Test						1.000

**Significant at the .01 level

(1) On the criterion of problem-solving ability, as measured by the Strategies Test, there was no significant difference between the IPI group and the control group.

(2) When adjustments were made for differences in intelligence and initial differences in problem-solving achievement, the control group achieved significantly higher scores than the IPI group on the criterion of problem-solving achievement as measured by the CTBS.

(3) When adjustments were made for differences in abilities, no significant differences between any of the IPI schools could be found when using either problem-solving ability or problem-solving achievement as the criterion.

(4) Problem-solving ability was found to be significantly related to problem-solving achievement and intelligence, and not related to age.

(5) No significant interaction was found between three intelligence levels and type of school when either problem-solving ability or problem-solving achievement was used as the criterion.

CHAPTER FIVE

SUMMARY, CONCLUSIONS, IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

I. SUMMARY

The purpose of this study was to compare the problem-solving abilities of grade six students in three IPI schools with the problem-solving abilities of grade six students in three control schools.

Thirteen grade six students were selected from each of the six schools to participate in the study. Each of the students selected had been enrolled in his respective school for the past two years. Students in the IPI schools during the last two years had been using IPI materials as a basis for their mathematics curriculum.

The Lorge-Thorndike Tests of Intelligence and the Problem-Solving Subtest of the Canadian Tests of Basic Skills had been administered to all students prior to the commencement of the IPI experiment. The grade six version of the CTBS was administered to all students in six schools in April, 1971. In addition a Strategies of Problem Solving Test was administered by means of an oral interview to the sample of 78 students during April and May, 1971.

On the basis of the data obtained, the achievement of the IPI sample was compared to the achievement of the control sample. The three IPI schools were then compared to each other. Correlations among the major variables were calculated. Interaction between three intelligence levels and both measures of problem-solving achievement was also tested.

II. CONCLUSIONS

Conclusions With Respect To Major Hypotheses

Hypothesis 1: On the basis of the analysis of hypothesis 1, the mathematics programs of the two types of schools seem to be equally effective in preparing students to solve unfamiliar arithmetic problems. The Strategies Test consisted of problems equally unfamiliar to all subjects in the study. The control schools, however, appear to be more effective in preparing students to solve problems on the CTBS standardized test.

The superiority of the control schools on the CTBS may be a result of a combination of factors. Students in the control schools may have developed a wider repertoire of algorithms which may be used in solving familiar problems. This is possible, because any number of students in the IPI schools may have spent a large amount of the last two years mastering concepts normally associated with previous grades. In this way, the problems on the CTBS normally associated with grade six arithmetic would appear novel to many of the IPI students. Another possibility is that the control students took the written test (CTBS) more seriously than did the IPI students. Tests are frequent in the IPI schools and may not be considered as threatening to IPI students as to control students. The oral test (Strategies Test) would have been equally novel to all students in the sample, hence no differences existed between the scores obtained by the two groups. The possibility also exists that the control schools have developed students with superior problem-solving abilities and that the Strategies Test is more an intelligence test than a means of measuring the process of problem solving.

Hypothesis 2: None of the IPI schools appeared to have achieved

significantly higher results on any of the tests than any other IPI school. As stated earlier in Chapter Three, Connors (1970) identified one of the schools as having a low socio-economic status, one of the schools as having a high socio-economic status, and one of the schools as being a rural school. With these facts in mind, socio-economic status does not appear too closely related to problem-solving abilities.

Hypothesis 3: Perhaps the least surprising finding of the study was that problem-solving ability is directly related to problem-solving achievement and intelligence. In a sample restricted to one grade level it is not surprising to find that age is not directly related to problem-solving ability, especially when age is found to be negatively related to intelligence.

Hypothesis 4: Neither type of school appears to give preferential treatment to any of the three intelligence levels identified in the study.

Other Conclusions

Grade six students engage in a variety of methods of problem solving, and they commit various errors when doing so. The nature of the problem and its familiarity to the student appear to influence the student's choice of method. If the problem seems familiar to a student he searches for an algorithm which he may use to solve the problem. If the problem seems very difficult or completely unfamiliar, a trial-and-error method is frequently resorted to. Trial-and-error, as a method of solution, was found to be used successfully by students of all ability levels.

Students in IPI schools appear to give more random responses when solving problems than students in the control schools. This conclusion is based on the low correlations among the problem-solving scores

and the intelligence scores. The fact that the IPI program does not have a specific unit on problem solving may lead to the development of problem-solving abilities on a somewhat incidental basis. The only IPI school to match the two-year net gain of the control schools on the CTBS was the IPI school that supplemented the IPI program with worksheets on computation and problem solving, some of which were assigned as homework. The other two IPI schools used worksheets much less frequently.

A subject's intelligence seems to be a better predictor of achievement, on a test composed of difficult and unfamiliar questions, than knowledge of the mathematics program to which the subject has been exposed. This conclusion is based on the fact that when the six schools were placed in a rank order for each variable, the order on the basis of the Strategies Test was identical to the order for the non-verbal I.Q. scores.

The high reliability obtained for the marking of the Strategies Test lends support to Affolter's (1970) claim that the problem-solving process involves three phases: sensing, predicting, and verifying. The processes used by grade six students can, at least, be reliably categorized under these three headings.

III. IMPLICATIONS

Several implications for the mathematics instruction of grade six children may be drawn.

The performance of grade six children on a standardized problem-solving test is enhanced by studying a formal unit of work on problem solving. The achievement on the CTBS indicated that the control students achieved higher scores than did the IPI students. Problem solving is

not included in the behavioral objectives outlined by the IPI program.

The results of the scores obtained on the Strategies Test imply that the nature of the particular arithmetic program does not greatly affect a student's ability to cope with unfamiliar problems.

These findings suggest that it is most desirable to provide grade six students with a wide variety of arithmetic problems to solve. Students who rely too heavily on a particular problem-solving algorithm find little success in coping with unfamiliar problems.

Although the test for interaction between problem-solving abilities and intelligence levels showed no interaction, the results did show that an individualized program is feasible for low ability students. The low ability students in the IPI group achieved higher, although not significantly higher, results on the Strategies Test than did the low ability students in the control group. On the basis of these results, individualized programs in arithmetic may prove to be most successful for the low ability student.

Having students explain in detail their method of solution proved to be a learning experience as well as a testing experience for most of the subjects in the study. This observation suggests that some of the difficulties grade six children experience in solving problems may be overcome by occasionally requiring students to verbally detail their solution to a problem.

With respect to the computational errors committed by the subjects in the study, grade six students require more practice in division of whole numbers.

IV. SUGGESTIONS FOR FURTHER RESEARCH

This study compared the problem-solving abilities of grade six

students in three IPI schools with the problem-solving abilities of grade six students in three control schools. The students in the IPI schools had been studying arithmetic from the IPI program for two years. A further study should be made in two or three years to investigate the effect that an IPI program has on students after they return to a conventional mathematics program.

A second recommendation is that if a test such as the Strategies Test is to be used to identify the processes used in problem solving, the subjects involved should be given preliminary training in thinking aloud as they solve problems. Such training would help minimize the interference that any verbalizing might have on the thought process.

A third recommendation is that some exploration is needed to determine whether students can be taught an efficient method of approaching problems which are unfamiliar to them.

Finally, some consideration should be given to developing a program such as IPI which does have specific objectives defining desireable problem-solving behaviors and contains at least one unit on problem solving.

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APPENDIX A
STRATEGIES OF PROBLEM SOLVING TEST

THE PROBLEMS (Revised, 1971)

I. "the lost ball"

Just for practice let's begin with a game called "Twenty Questions." Here is a chart of a field (manilla tag marked off into 132 one-inch squares - 11 X 12). Let's say that a boy lost his baseball somewhere in this field. I know where it is, but you do not. You have twenty questions to see if you can find out where it is.

II. "the telephone call"

Sunday telephone calls from Penhold Air Force Base to Bob's home cost 60¢ for the first 3 minutes, and 15¢ for each additional minute. Bob called home on Sunday afternoon. He talked for 7 minutes. How much did the call cost?

III. "how much copper"

A nickel (five cents) is a mixture of both nickel and copper. For every pound of nickel that goes into the mixture, there are three pounds of copper. How many pounds of copper would be needed to make 24 pounds of the nickel coins?

IV. "the measurement?"

Joe threw a ball. Then he laid a 72-inch tape down 23 times to see how far he had thrown the ball. Jim threw a ball 118 feet. Who threw the ball farther? How much farther?

V. "the vinegar problem"

A supermarket bought 275 gallons of vinegar in large barrels which had to be emptied right away and sent back to the factory. The manager of the supermarket found that he had only 168 empty gallon jars that he could fill with vinegar. He knew that these would not be enough to hold all the vinegar, so he decides to put the rest of the vinegar into pint containers. Could you figure out how many pint containers he will need?

VI. "father's age"

A man married at the age of 25. His wife died 15 years after

their marriage, leaving a daughter who was only 11 years old. After 9 more years, the daughter married a man who was 4 years older than she was. When her father died, the daughter's husband was 45 years old. Is there any way that you could figure out how old her father was when he died?

VII. "gift of money"

Two brothers, Floyd and David, each Christmas receive a gift of money from their uncle, to be divided equally between them. The total amount of money that their uncle sends for both of them is always equal to the product of their ages (their ages multiplied together). This past Christmas, in 1970, their uncle sent $36. The Christmas before last, in 1969, he sent $22 for the boys to divide between them. Could you figure out how old the boys were last Christmas, that is 1970?

THE PROBLEMS (Affolter, 1970)

I. "the lost ball"

Just for practice let's begin with a game called "Twenty Questions." Here is a chart of a field (manilla tag marked off into a 132 one-inch squares - 11 X 12). Let's say that a boy lost his baseball somewhere in this field. I know where it is, but you do not. You have twenty questions to see if you can find out where it is.

II. "how many socks"

Jane was getting dressed for a school concert when a thunderstorm caused all the lights in the house to go out before she had picked out a pair of matching socks from her drawer. Her father is waiting in the car to drive her to the school. They have just enough time to get there before the concert starts. She knows that she has only yellow, white and blue socks in her drawer and that any of these colors will match the dress she is wearing. How many socks should Jane take with her so that she can be sure of having two socks the same color to put on in the car? (Explain that the socks are not arranged in pairs in the drawer. For boys in the study Jane became "John" and the colors were blue, green and brown.)

III. "how much copper"

A nickel (five cents) is made of both nickel and copper. For every pound of nickel that goes into the mixture, there are three pounds of copper. How many pounds of copper would be needed to make 24 pounds of the nickel coins?

IV. "chess and checkers"

All of the students in a certain class know how to play either chess or checkers, but some know how to play both games. There are 14 students who know how to play chess; there are 16 students who can play checkers; but there are 10 students who can play both chess and checkers. Can you figure out how many students are in the class?

V. "the vinegar problem"

A supermarket bought 275 gallons of vinegar in large barrels which had to be emptied right away and sent back to the factory. The manager of the supermarket found that he had only 168 empty gallon jars that he could fill with vinegar. He knew that these would not be enough to hold all the vinegar, so he decides to put the rest of the vinegar into pint containers. Could you figure out how many pint containers he will need?

VI. "father's age"

A man married at the age of 25. His wife died 15 years after their marriage, leaving a daughter who was only 11 years old. After 9 more years, the daughter married a man who was 4 years older than she was. When her father died, the daughter's husband was 45 years old. Is there any way that you could figure out how old her father was when he died?

VII. "gift of money"

Two brothers, Floyd and David, receive a gift of money each Christmas to be divided equally between the two boys. The total amount of money that their uncle sends for both of them is always equal to the product of their ages (explain the meaning of product). This past Christmas, in 1969, the uncle sent $36. The Christmas before last, in 1968, he sent $22 for the boys to divide between them.

Could you figure out how old the boys were last Christmas; that is what were their ages in 1969 when the uncle sent them $36?

QUESTIONS AND COMMENTS MADE BY THE EXPERIMENTER

1. You may ask any questions you like, however, I may not permit myself to answer some questions.

2. If you feel that you need help with computation, you may ask me for assistance.

3. Could you tell me how you got that answer?

4. Why did you multiply (or divide, or whatever) those numbers?

5. What does that number mean?

6. What were you trying to find when you subtracted (or added, or whatever)?

7. Do you think your answer is right?

8. Are you sure your answer is right?

9. Is there any way you can check your answer?

CRITERIA FOR SCORING STRATEGIES TEST

PROBLEM II "THE TELEPHONE CALL"

Sensing:	Score
a. Recognizes that the 3 minutes is included in the total time of 7 minutes and that there are two different rates involved.	2
b. Has only a partial grasp of the situation, but does see that there are two different rates.	1
c. Does not understand the structure of the problem; sees no point in including all the numbers.	0
Predicting:	
a. Reaches the correct solution by subtracting 3 from 7 and then proceeding with muliplication and addition.	2
b. Multiplies 7 by 15, and then adds the cost of the first 3 minutes has been charged for twice, or doesn't see the word "first" and takes 60¢ for 3 minutes, 60¢ for the next 3 minutes, and 15¢ for 1 additional minute.	1
c. Randomly adds, mulitplies numbers.	0
Validating:	
a. Gives a valid reason for subtracting 3 from 7 and for multiplying by 15.	2
b. Simply restates his method, not seeing any error in it.	1
c. Gives no reason for the computation used. Is confident that his answer is correct because his computation is correct.	0

PROBLEM III "HOW MUCH COPPER"

Sensing:	Score
a. Student shows that problem situation is firmly grasped and structured to permit search for a solution: e.g. "The 24 lbs. of coins is made up of nickel and copper both; the recipe says 1 lb. of nickel to 3 lbs. of copper" or "I have to find out how much of this 24 lbs. is copper."	2
b. Situation is only partially grasped; subject doesn't recognize that the 24 lbs. is made up of nickel and copper both, but knows that there has to be three times as much copper as there is nickel.	1
c. Subject randomly manipulates numbers without showing that he understands that the amount of copper must be three times that of nickel.	0
Predicting:	
a. Arrives at the correct answer, 18 lbs. of copper either by use of rate equation, deduction or some kind of approximation.	2
b. Suggests only inappropriate solutions but has shown that the problem is understood.	1
c. Refuses to predict a solution; says it can't be done.	0
Validating:	
a. Gives a valid reason for a correct solution: e.g. "18 is 3 times 6 and 18 and 6 together come out to 24 lbs. so it has to be right.	2
b. Recognizes that 8 lbs. or 72 lbs. can't be correct because of a valid reason; but fails to come up with the correct solution.	1
c. Makes no attempt to validate a solution; says it has to be riht because 24 times 3 is 72 or 24 divided by 3 is 8: relies on correctness of computation to verify an answer.	0

PROBLEM IV "THE MEASUREMENT"

Sensing:	Score
a. Recognizes the problem as requiring the changing of inches to feet or feet to inches somewhere in the problem, and that subtraction is necessary to compare the two distances.	2
b. Understands that it is necessary to multiply by 23, but does not recognize the need to change units of measure.	1
c. No grasp of the problem; what is given and what is required.	0
Predicting:	
a. Successfully uses all the information to arrive at a solution.	2
b. Fails to arrive at a correct answer because of a small error in method or computation.	1
c. Randomly computes with the given numbers, failing to come to grips with the problem.	0
Validating:	
a. Gives adequate justification for believing 20 feet, or 240 inches is the correct answer; that laying a rule down 23 times means that you have to multiply by 23, and that units can only be compared if they are identical.	2
b. Gives only a partially acceptable explanation for correctness or recognizes that his answer is wrong, but cannot find a different way to do it.	1
c. Has no understanding of the problem; relies on accuracy of computation to verify his solution.	0

PROBLEM V "THE VINEGAR PROBLEM"

Sensing:	Score
a. Recognizes the problem as requiring subtraction to find the shortage of containers and that the remainder will be in gallons and must be changed into pints.	2
b. Partially grasps the problem situation but fails to see the necessity of changing the units.	1
c. Doesn't accept it as a problem; says he can't do it.	0
Predicting:	
a. Carries out all the necessary calculations to arrive at the correct solution.	2
b. Only does part of the required calculations or makes an error in computation or a small error in methodology.	1
c. Manipulates numbers in a random fashion without coming to grips with the problem.	0
Validating:	
a. Gives a good reason for believing that 856 pints is the correct answer by explaining the necessity of subtracting and changing gallons to pints. Gives adequate explanation and realizes that the only way the answer could be wrong is if they used 16, or whatever instead of 8 pints in a gallon.	2
b. Gives a poor reason for a correct solution: e.g. the correctness of the computation; or recognizes that an answer is incorrect but is unable to find another solution.	1
c. If the subject has engaged in random calculations showing no grasp of the problem assign 0 for this section also.	0

PROBLEM VI "FATHER'S AGE"

Sensing:	Score
a. Grasps the significance of the relationship between the son-in-law's age and the age of the father.	2
b. Only partially grasps the significance of the son-in-law's age as it relates to the age of the father.	1
c. No grasp of the problem; what is given and what is required.	0
Predicting:	
a. Arrives at the correct answer, 70 years, by appropriate calculations.	2
b. Omits to do part of the calculation or makes a computational error.	1
c. Randomly adds numbers without understanding the problems.	0
Validating:	
a. Gives a good reason for adding the 21 years to 49: e.g. 45 minus 24 is the number of years they were married; the father lived all that time too, so he must have been 21 years older too.	2
b. Partially validates the solution: e.g. gives reasons for bringing the father's age to 49 but is unable to tell why the 21 years should be added.	1
c. Gives no reasons for adding numbers in a random fashion.	0

PROBLEM VII "THE GIFT"

Sensing:	Score
a. Grasps the significance of the two amounts of money and understands what is asked for; also realizes that there is a relationship between the amount of money that is sent for the two consecutive years: e.g. "I have to find two numbers that go into 36 evenly and when you subtract one from each of them and then you multiply you'll get 22."	2
b. Partially grasps the problem; realizes the significance of finding two factors of 36 and 22 but doesn't see the relationship of the amounts in terms of two consecutive years.	1
c. No grasp of the problem at all.	0
Predicting:	
a. Searches for and finds the correct combination of ages which will accommodate the one-year difference in their ages.	2
b. Suggests several combinations of ages, which are factorizations of 36, but is unable to reach the correct solution.	1
c. Says that there is just no way of figuring the boy's ages when all you know is the money they received.	0
Validating:	
a. Gives a good reason for believing that 3 and 12 are the right answers: e.g. 3 times 12 gives you 36 dollars for 1970 and then if you take one year off each age and multiply you get 22."	2
b. Gives a valid reason for believing that suggested combinations are not right, but is unable to find the correct solution.	1
c. Is unable to find any way of checking to see if a combination that works for 1970, such as 9 and 4, are actually the ages that the boys were.	0

APPENDIX B

RAW DATA

TABLE XX

DATA PERTAINING TO STUDENTS FROM CONTROL SCHOOLS

Student number	Sex	Age in months	Verbal I.Q.	Non-verbal I.Q.	Strategies Test						CTBS 1969	CTBS 1971
					No. II	III	IV	V	VI	VII		
202	F	143	119	113	4	0	6	3	0	0	16	25
206	M	138	106	116	6	3	3	5	4	2	16	16
207	M	144	101	105	6	1	3	3	6	0	12	13
210	M	152	94	99	3	1	3	2	3	0	14	12
211	F	148	123	130	3	1	5	4	6	0	21	24
213	M	140	112	120	2	1	5	6	1	0	11	24
217	M	171	75	84	4	1	3	6	0	0	8	14
226	M	131	122	130	6	1	6	6	6	2	9	28
229	F	143	127	141	6	1	6	6	6	6	21	27
231	F	144	95	98	2	0	0	2	0	0	10	12
233	F	146	146	134	4	1	3	6	6	6	16	26
234	M	141	87	108	6	1	6	6	3	3	12	23

TABLE XX continued

239	F	142	112	106	6	1	3	6	3	4	18	23
401	F	148	125	134	6	1	6	6	3	0	19	27
404	F	145	99	123	6	1	6	5	3	6	18	21
405	F	145	96	93	1	0	0	2	0	0	6	13
407	F	137	97	104	0	0	0	2	0	0	4	13
408	M	163	81	84	0	0	4	4	6	2	5	13
409	F	143	108	128	2	1	2	6	3	0	17	28
410	M	142	86	109	6	3	3	2	3	0	6	7
413	M	144	105	115	6	1	3	6	5	0	16	21
416	M	140	125	106	6	4	3	3	6	0	12	14
418	M	143	109	116	3	0	6	6	6	3	14	21
419	F	148	92	102	4	1	3	0	3	0	7	18
420	F	146	107	126	6	1	3	4	6	6	11	22
421	F	145	112	113	2	1	3	3	5	6	17	20
608	F	146	105	97	6	0	3	5	3	0	14	19
612	F	139	103	105	2	1	3	3	3	0	15	19
614	F	140	113	123	6	1	6	6	5	0	14	18

TABLE XX continued

617	M	150	81	111	6	0	6	3	3	0	12	17
618	M	157	92	98	3	1	6	4	2	0	17	19
619	M	156	72	75	1	0	0	2	0	0	6	16
631	F	141	88	100	3	0	3	2	3	0	15	22
638	F	141	109	126	3	1	3	2	6	0	21	24
643	F	146	116	130	6	1	6	6	6	6	18	28
650	M	155	96	83	2	1	2	5	6	0	10	11
652	M	139	89	90	4	0	6	3	0	0	10	12
655	M	140	104	117	6	1	6	3	5	0	19	22
658	M	148	111	115	3	1	6	5	6	3	18	23

TABLE XIX

DATA PERTAINING TO SUBJECTS FROM IPI SCHOOLS

Student number	Sex	Age in months	Verbal I.Q.	Non-verbal I.Q.	Strategies Test						CTBS 1969	CTBS 1971
					No. II	III	IV	V	VI	VII		
101	M	144	96	107	3	0	3	3	3	0	13	8
107	F	146	107	121	1	0	0	3	6	0	16	15
108	F	143	82	98	6	1	3	6	3	0	6	13
115	F	138	114	122	1	1	4	3	0	4	7	14
123	M	142	115	128	6	6	5	6	6	6	15	26
124	M	143	80	90	6	1	6	6	6	2	6	10
125	M	139	126	120	6	0	3	3	5	0	18	15
126	F	139	106	116	3	6	5	5	4	0	10	5
127	M	139	116	122	6	1	5	6	5	6	12	20
136	F	140	99	120	1	0	6	5	0	0	8	12
137	F	133	129	126	6	0	5	5	6	0	16	13
138	F	134	82	102	3	0	3	0	3	0	13	6
141	M	142	119	131	6	1	6	5	6	6	12	19

TABLE XIX continued

301	M	143	120	107	6	1	6	3	6	0	15	21
303	M	145	103	111	5	6	4	3	6	0	7	10
304	F	145	114	115	6	1	6	5	0	0	11	22
306	M	147	96	105	4	0	3	6	5	0	13	9
307	F	148	96	113	5	1	3	3	6	0	7	20
308	M	136	95	101	4	0	0	0	6	0	6	10
310	F	145	103	126	6	1	5	3	6	0	17	20
311	F	137	109	111	4	0	6	6	5	0	7	11
312	M	144	91	102	4	2	6	0	5	3	7	22
313	F	139	98	93	3	1	0	0	0	0	6	7
314	F	143	113	108	6	1	3	3	2	6	10	21
315	M	139	109	112	6	1	4	5	6	0	6	21
316	M	152	88	78	3	0	0	2	0	0	8	10
504	F	141	131	120	3	1	3	3	6	0	6	13
505	F	138	107	90	2	4	4	3	3	0	9	13
507	F	143	108	105	6	0	3	3	3	0	12	17
514	F	142	91	100	6	1	3	5	3	0	12	11

TABLE XIX continued

515	F	138	130	126	6	0	3	4	0	0	12	17
517	F	142	101	102	6	0	3	5	5	0	10	17
523	F	136	107	83	6	1	3	2	0	0	7	8
527	M	158	71	81	6	1	2	6	6	0	4	4
528	M	136	109	103	1	1	1	4	1	2	8	11
537	M	145	101	96	2	0	3	3	0	0	12	16
546	M	141	104	107	3	3	6	6	0	0	11	9
547	M	145	89	100	6	6	3	6	6	0	9	8
551	M	141	106	82	6	1	3	2	6	0	7	16